A-Level
Computer Science

Logic gates and circuits

Lesson Objectives

Students will learn:

- Logic gates (NOT, AND, OR, NAND, NOR & XOR).
- To apply the concept of logic gates to analyse and build logic circuits.
- To design and analyse logic circuits.

1.

Content

What are logic gates?

- Logic gates are the building blocks of electronic circuits that are used in computer components like memory and other controlling devices.
- Logic gates work on the principle that the binary digit 1 represents the ON or TRUE state and 0 represents the OFF or FALSE state.

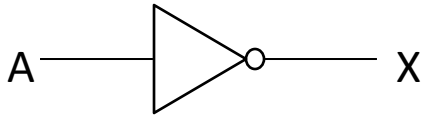
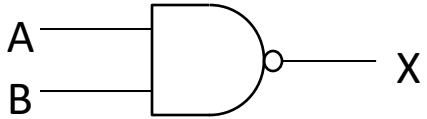
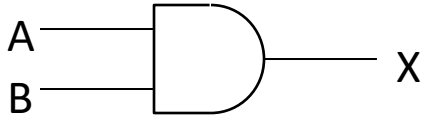
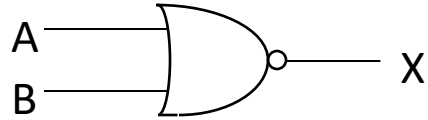
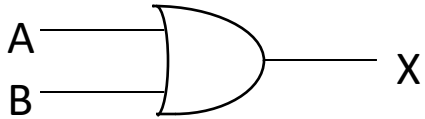
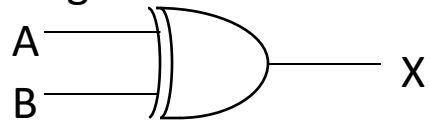
Why do we need logic gates?

- A computer understands the binary language. So, the components of the computer contain logic gates, which work on the binary system.
- The data and instructions are processed in binary form.

Computers are composed of nothing more than logic gates stretched out to the horizon in a vast numerical irrigation system.

-Stan Augarten, in State of the Art: A Photographic History of the Integrated Circuit, 1983

Logic gates

NOT gate 	NAND gate 
AND gate 	NOR gate 
OR gate 	XOR gate 

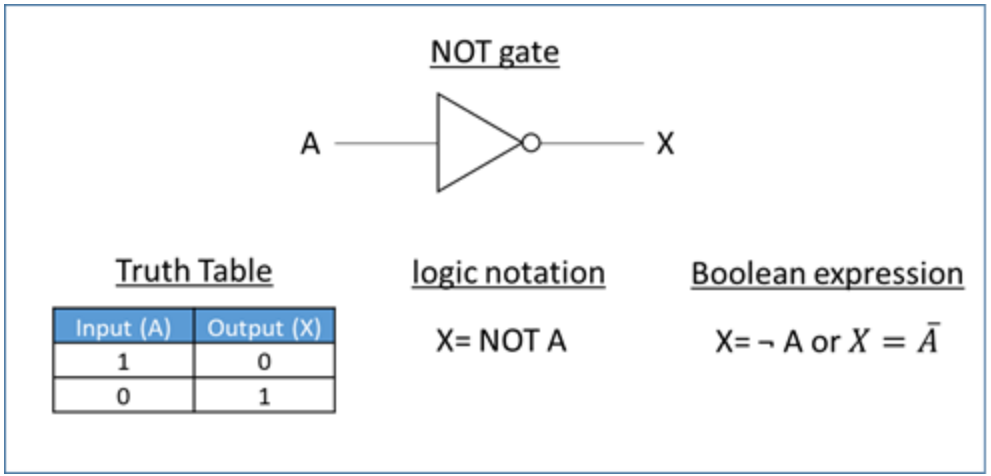
Truth table

- A truth table is used to denote the different outputs of the logic gates or circuit with respect to different inputs.
- A NOT gate has one input and, hence, has 2^1 possible combinations.
- The OR and AND gates have two inputs and 2^2 output combinations are possible.
- Logic circuits can also have more than two inputs. For n inputs, the number of possible output combinations is 2^n.

Number of possible Inputs	Number of possible Outputs
1	**$2^1 = 2$**
2	**$2^2 = 4$**
n	**2^n**

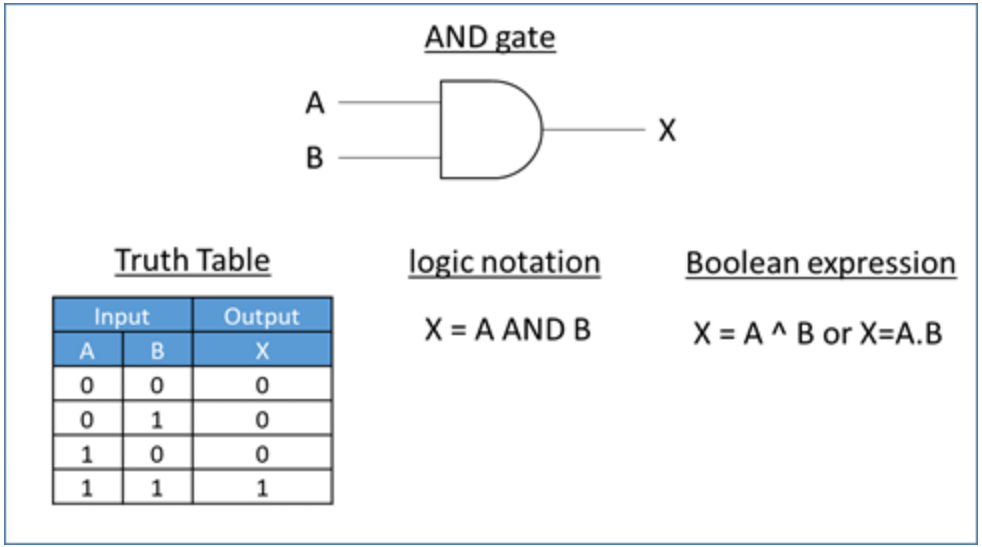
Logic gates: NOT gate

- The output of the NOT gate is complementary to the input.



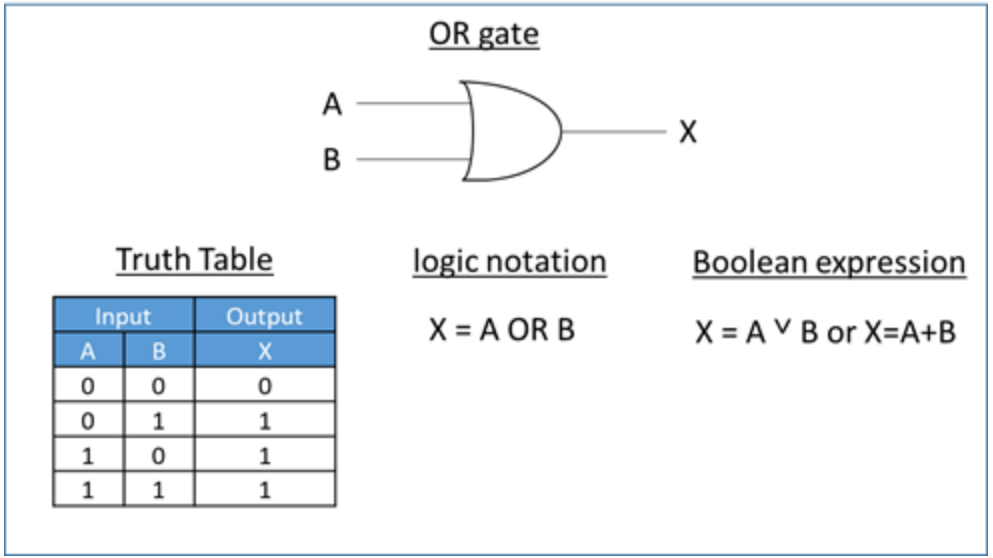
Logic gates: AND gate

- The output of an AND gate is 1 only when both the inputs are 1.



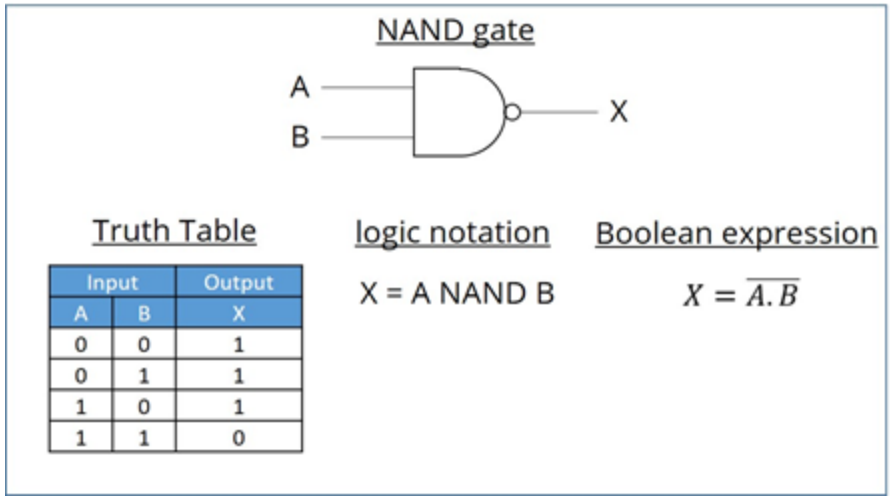
Logic gates: OR gate

- The output of an OR gate is 1 when any one of the inputs is 1.



Logic gates: NAND gate (NOT AND gate)

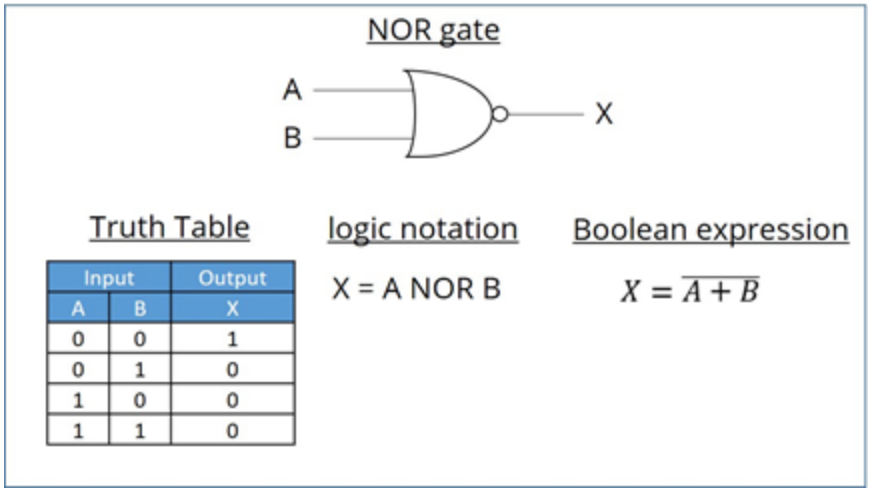
- NAND gate is a combination of AND and NOT gate.
- The output of a NAND gate is complementary to that produced by an AND gate.



Input		Output
A	B	X
0	0	1
0	1	1
1	0	1
1	1	0

Logic gates: NOR gate

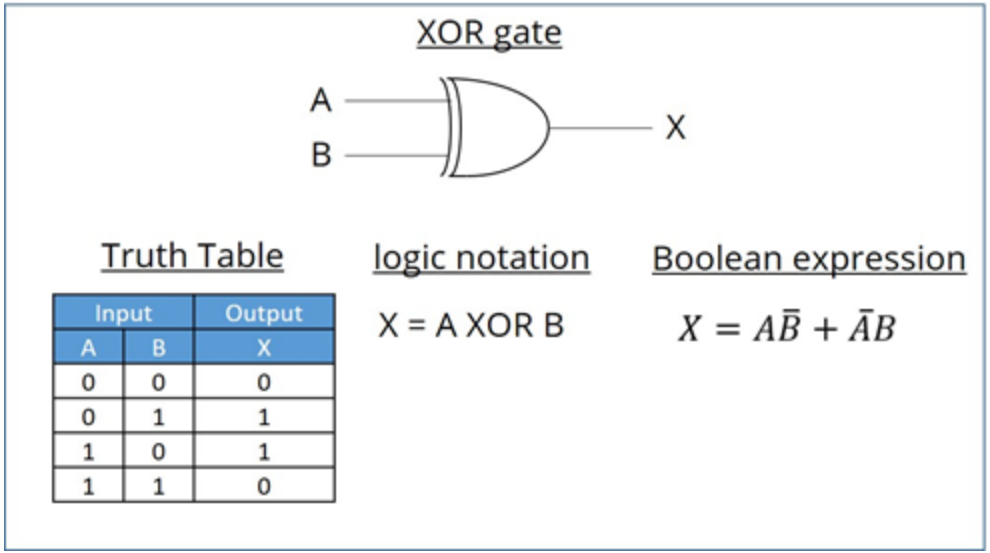
- NAND gate is a combination of OR and NOT gate.
- The output of a NAND gate is complementary to that produced by an OR gate.



Input		Output
A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

Logic gates: XOR gate

- The output of an XOR gate is 1 when both inputs A and B have different values.



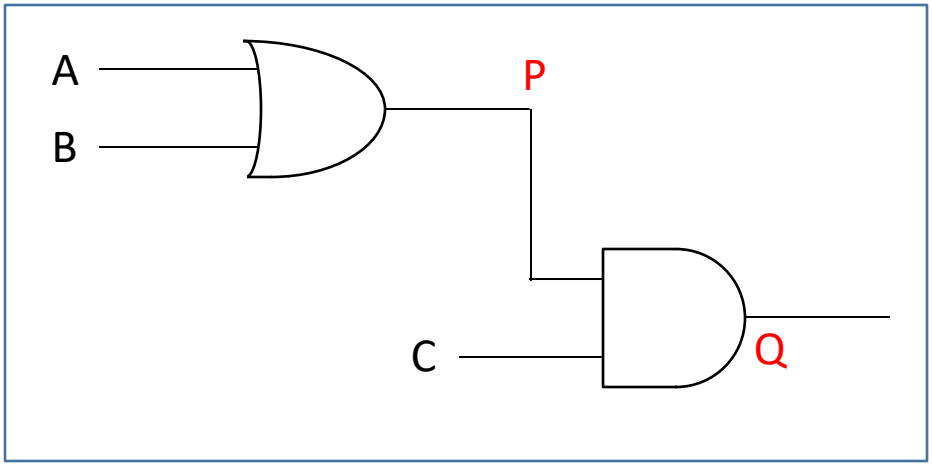
Input		Output
A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

What are logic circuits?

- Logic gates are combined to form logic circuits that are responsible for a unique function, for example, controlling the various mechanisms of an oven such as temperature, timing, etc.

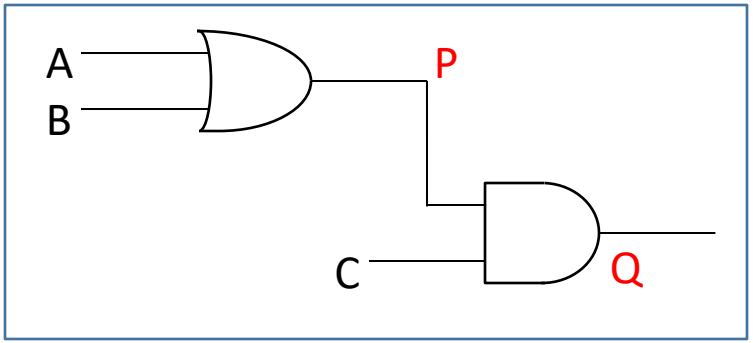
Logic circuits: Example I

- Let us consider the following circuit. We will analyse the circuit and determine its truth table. It can be noted that there are three inputs, hence, 2^3=8 possible binary combinations.



Analysing logic circuits: Step 1

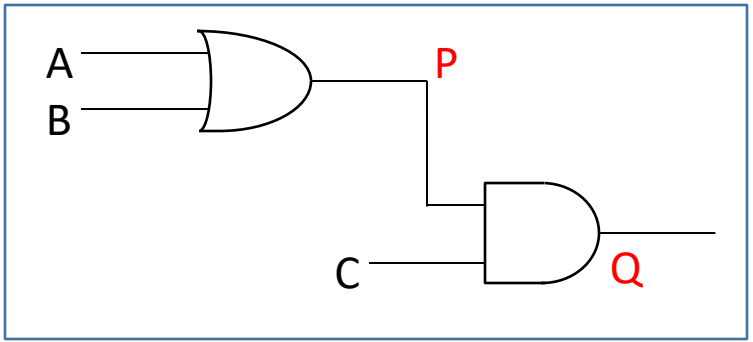
- To reduce errors, the circuit is split into two parts with an intermediate output P and final output Q.
- P is the output of the first OR gate with inputs A and B. P is true when either A or B is true.



Input			Intermediate output
A	B	C	P (A or B)
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

Analysing logic circuits: Step 2

- Q is the output of the AND gate with P and C as inputs. Q is true only when both P and C are true.

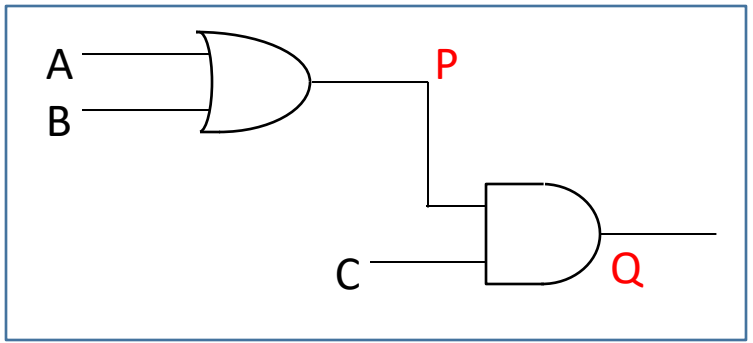


Input			Intermediate output	Final output
A	B	C	P (A or B)	Q (P AND C)
0	0	0	0	0
0	0	1	0	0
0	1	0	1	0
0	1	1	1	1
1	0	0	1	0
1	0	1	1	1
1	1	0	1	0
1	1	1	1	1

Analysing logic circuits: Step 3

- The intermediate output is removed to obtain the final truth table.
- Using the logic circuit, the logic notation of output Q is Q= (A OR B) AND C. Using symbols in logical notation,

$$Q = (A + B).C$$

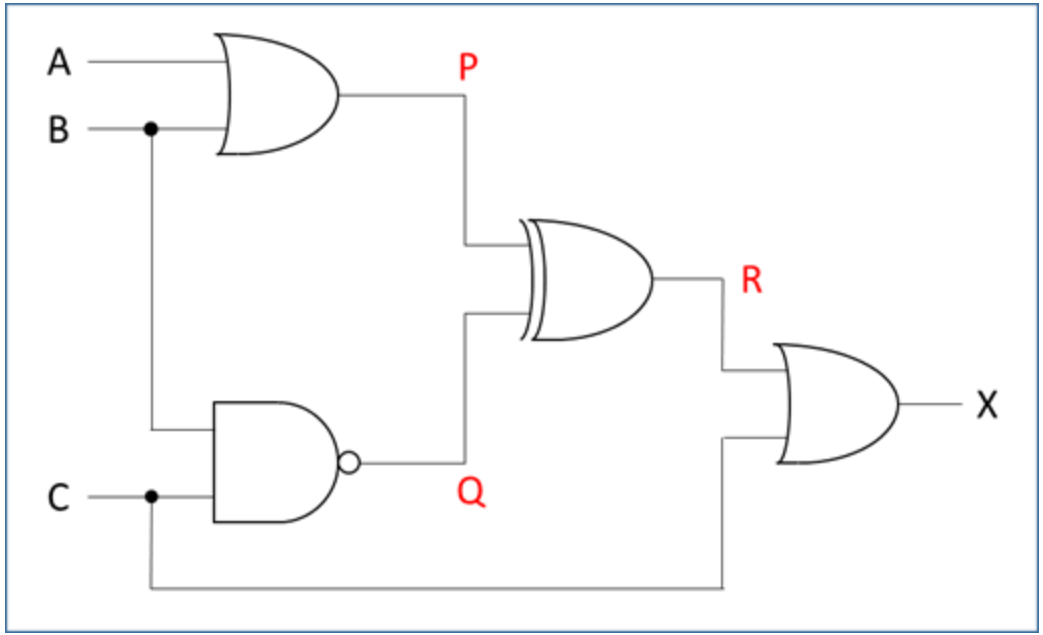


Input			Final output
A	B	C	Q (P AND C)
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Logic circuits: Example II

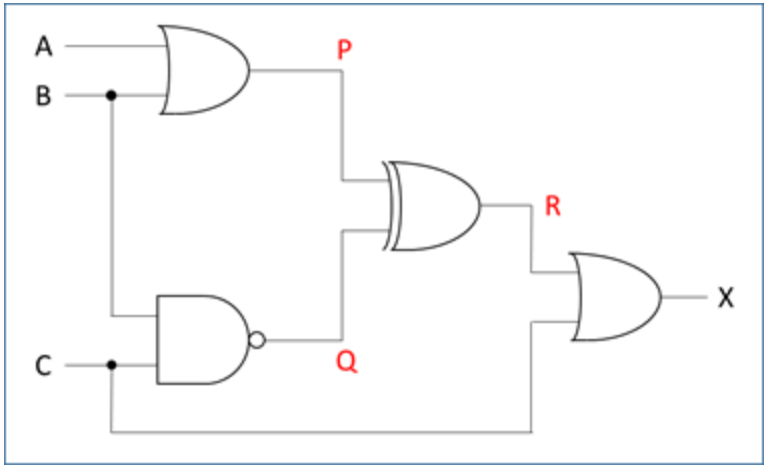
Another logic circuit is shown in the figure below.

Again, there are three inputs and 8 possible binary combinations.



Analysing logic circuits: Step 1

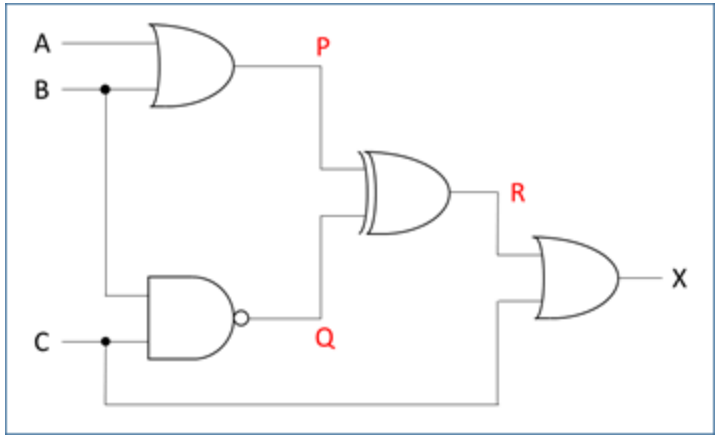
- The circuit is split into three parts with outputs P, Q and R.



Input			Intermediate output	
A	B	C	P (A+B)	$\overline{Q}$ $\overline{(B.C)}$
0	0	0	0	1
0	0	1	0	1
0	1	0	1	1
0	1	1	1	0
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	0

Analysing logic circuits: Step 2

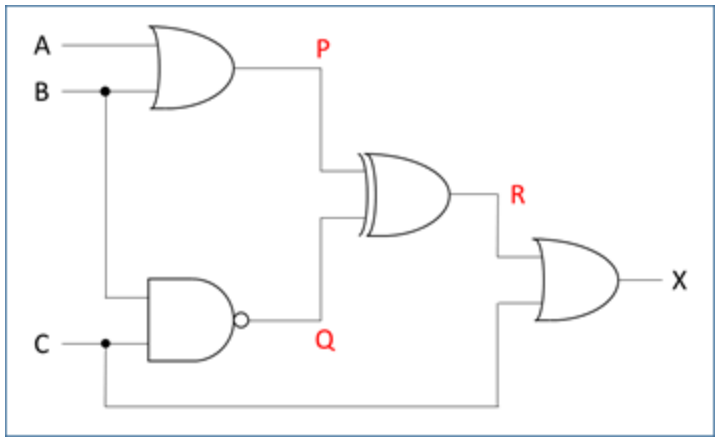
- R is the output of the XOR gate where the inputs are P and Q.



Input			Intermediate output		
A	B	C	P	Q	$R(P\overline{Q}+\overline{P}Q)$
0	0	0	0	1	1
0	0	1	0	1	1
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	1	0
1	0	1	1	1	0
1	1	0	1	1	0
1	1	1	1	0	1

Analysing logic circuits: Step 3

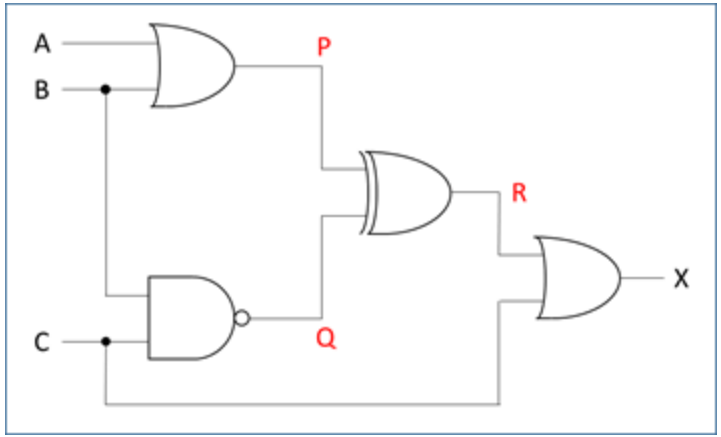
- The final output X is the output of an OR gate where R and C are inputs. The input for C is taken from the table 1.



R	C	X
1	0	1
1	1	1
0	0	0
1	1	1
0	0	0
0	1	1
0	0	0
1	1	1

Analysing logic circuits: Step 4

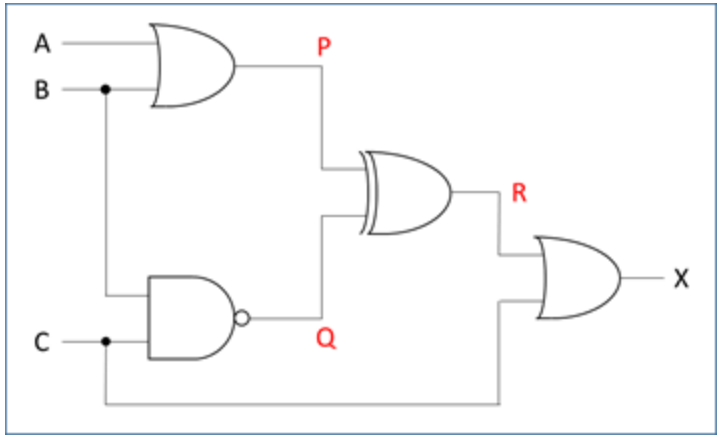
- Combining the intermediate outputs,



Input			Intermediate output			Final o/p
A	B	C	P	Q	R	X
0	0	0	0	1	1	1
0	0	1	0	1	1	1
0	1	0	1	1	0	0
0	1	1	1	0	1	1
1	0	0	1	1	0	0
1	0	1	1	1	0	1
1	1	0	1	1	0	0
1	1	1	1	0	1	1

Analysing logic circuits: Step 5

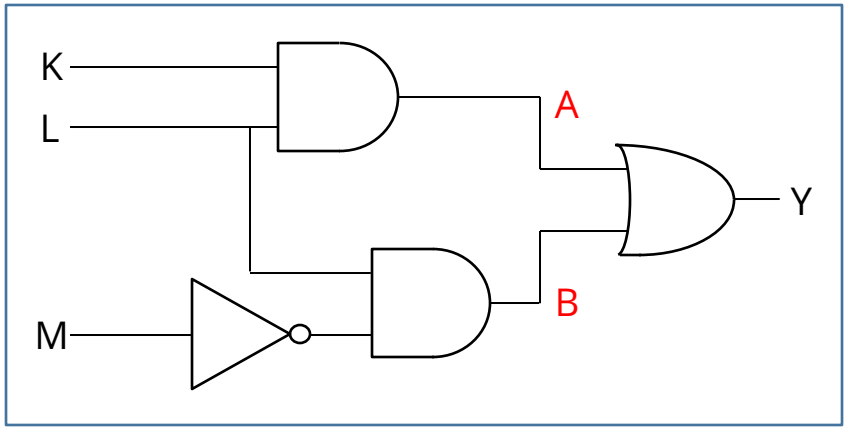
- The intermediate outputs are removed to obtain the final truth table,



Input			Final output
A	B	C	X
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

Designing a logic circuit

- A safety system has three inputs K, L and M. An alarm Y, sounds if input K is ON and L is ON; or if input L is ON and M is OFF.
- The logical statement for the function of Y is Y= 1 if (K=1 AND L=1) OR (L=1 AND M=NOT 1).

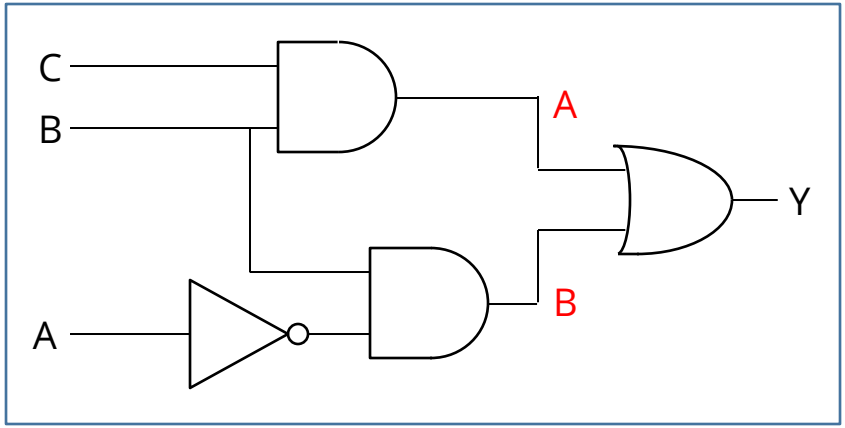


$\Rightarrow$ Y = (K AND L) OR (L AND NOT M)

$\Rightarrow$ Y = K.B + L. ¬ M

Designing a logic circuit: truth table

Input			Int. outputs		Final output
K	L	M	A	B	Y
0	0	0	0	0	0
0	0	1	0	0	0
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	1	1	1
1	1	1	1	0	1



The truth table can be cross-checked with a logic circuit and logic statement

Y= 1 if (K=1 AND L=1) OR (L=1 AND M=NOT 1)

Let’s review some concepts

Logic gates

Logic gates are the building blocks of electronic circuits that are used in computer components like memory and other controlling devices.

Truth table

For n inputs, the number of possible output combinations is 2^n.

NOT gate

The output of the NOT gate is complementary to the input.

AND gate

The output of an AND gate is 1 only when both the inputs are 1

OR gate

The output of an OR gate is 1 when any one of the inputs is 1.

Logic circuits

Logic gates are combined to form logic circuits that are responsible for a unique function, for example, controlling the various mechanisms of an oven such as temperature, timing, etc.

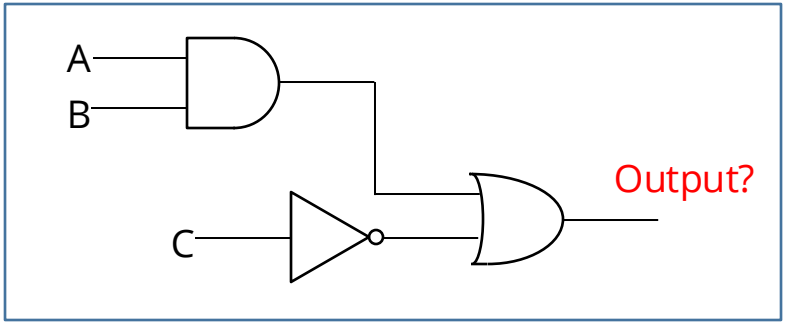
2.

Activities

Activity-1

Duration: 10 minutes

A logic circuit is given. Find out the output of this circuit by filling in the truth table. Also, write down the logical statement for final output in the space given.



Activity-2

Duration: 15 minutes

A logic circuit is given. An alarm, X, sounds if input A is OFF and B is ON; or if input B is OFF and C is OFF. Find out the output of this circuit by filling in the truth table.

a) What is the logical statement for alarm X to sound?

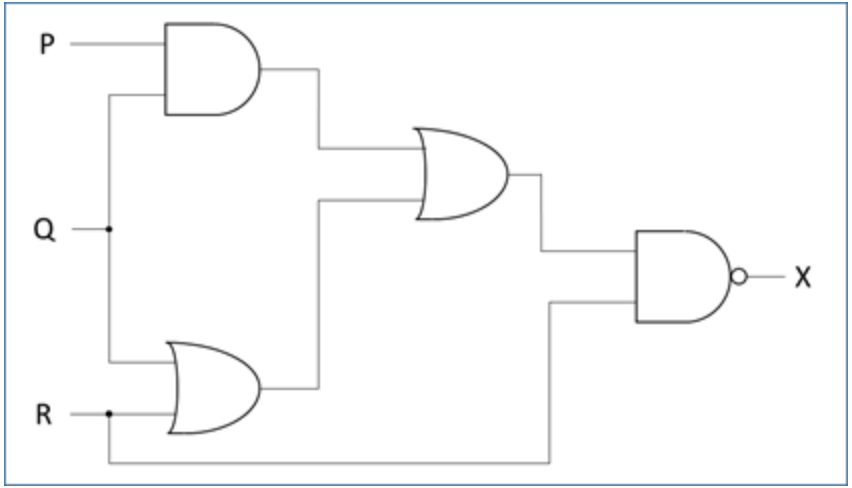
b) Draw the logic circuit for function X.

c) Fill in the truth table for this logic circuit and cross-check your answer.

Activity-3

Duration: 15 minutes

A logic circuit is given. Find out the output of this circuit by filling in the truth table. Also, write down the logical statement for final output in the space given.



3.

End of topic questions

End of topic questions

1. Draw the symbols for the fundamental logic gates (NOT, OR and AND gates).
2. Write down the truth table for the gates specified in question 1.
3. For the logic gates in question 1, write down the Boolean expression using appropriate symbols.
4. Draw the symbol for NAND gate. Write down its truth table and Boolean expression.

End of topic questions

5. Write down the truth table for the Boolean expression $R = P\bar{Q} + \bar{P}Q$. What logic gate is used to represent this expression?
6. A logic circuit has 3 inputs. How many possible output combinations are possible?
7. Derive the truth table for P = (A AND (NOT B)).

End of topic questions

9. Draw the following logic circuit:
   a) $\overline{(A + \bar{B})}$
   b) $(A + \overline{B}) . C$
   c) $A + \overline{B . C}$
10. Derive the truth table for the logic circuits in question 9.

End of topic questions

11. Create the truth table for the logic circuit.